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Technical Notes

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Effect of Laser Pulse Duration on Particle Image Velocimetry

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Nomenclature

b	=	bias error in pixels
d_x	=	$d_\tau + \delta$, major axis of the particle image (along x axis)
d_y	=	d_τ, minor axis of the particle image (along y axis)
d_τ	=	particle image diameter in pixels (without streaking)
e	=	d_x/d_y, eccentricity of the particle image
I	=	image intensity
k	=	slope of bias error vs displacement line
M	=	magnification of the experimental setup
R	=	$\tau/\Delta t$, ratio of laser pulse width to laser pulse separation used in particle image velocimetry
U	=	velocity along the x direction
x, y, z	=	Cartesian axes
Δt	=	laser pulse separation
ΔX	=	pixel displacement along x axis
ΔY	=	pixel displacement along y axis
δ	=	particle streak length
τ	=	laser pulse duration

Introduction

PARTICLE-IMAGE-VELOCIMETRY (PIV) measurements are typically made with flashlamp-pumped Nd:YAG lasers whose short pulses (about 10 ns) are usually sufficient to freeze the motion of the seed particles even in hypersonic flows. In contrast, diode-pumped acousto-optically Q-switched Nd:YAG and Nd:YLF lasers, which are seeing increasing use for cinematographic PIV, deliver pulses that are hundreds of nanoseconds in duration. When these relatively long-pulse lasers are used for PIV in supersonic flows, the particle images can appear as streaks elongated in the predominant flow direction rather than circular.[1] The presence of particle streaking can result in additional uncertainty in the velocity when conventional PIV processing algorithms are used.

Particle streaking has been known to be an issue for several years, particularly in studies that used video-rate cameras and continuous-wave lasers. For example, Willert and Gharib[2] performed PIV experiments in low-speed flows using an argon-ion laser and a video camera. This arrangement caused the particle images to appear as streaks; however, because of the low-speed nature of the flow they argued that this did not affect the accuracy of their measurements. Furthermore, Raffel et al.[3] suggest that the exposure time should be less than a quarter of the duration of a video frame to avoid excessive streaking of particles.

In this Note we present a study that is aimed at determining the effect of particle-image elongation on the accuracy of PIV. This is accomplished by generating synthetic particle fields with noncircular particle images and processing these fields with a conventional PIV algorithm. This work is most relevant to PIV in high-speed flows where streaking cannot be avoided, and the results should enable PIV users to design experiments that will minimize the errors induced by streaking.

Discussion

Particle streaking is a function of the local flow velocity and the laser pulse duration. For simplicity, consider a flow predominant along the x axis. The particle streak length δ in pixels is given by

$$\delta = U\tau/M \tag{1}$$

where

$$U = M\Delta X/\Delta t \tag{2}$$

Therefore,

$$\delta/\Delta X = \tau/\Delta t \tag{3}$$

Particle streak length is a function of the ratio of pulse duration to the pulse separation ($R = \tau/\Delta t$) and the pixel displacement (ΔX). Note that the size of the particle images (i.e., area occupied on the imaging array) also changes with these same parameters, and this has a direct effect on PIV accuracy.

In the current study, the synthetic particle images are described by a Gaussian intensity profile:

$$I(x, y) = I_o \exp\left[8(x - x_o)^2\big/d_x^2\right] \exp\left[8(y - y_o)^2\big/d_y^2\right] \tag{4}$$

where

$$I_o = I_m \exp\left[-8z_o^2\big/\delta z^2\right] \tag{5}$$

and (x_o, y_o, z_o) is the randomly generated location of a particle centroid within the field of view. Variation in z_o is used to simulate the variation in peak intensity of the particle images. Here d_x and d_y are the image diameters along the x and y axes, respectively, and δz is the thickness of the laser sheet. In the example considered in this study where the flow is predominantly along the x axis, the particle size would be

$$\begin{aligned} d_y &= d_\tau \\ d_x &= d_\tau + \delta \\ &= d_\tau + f(\Delta X, R) \end{aligned}$$

where d_τ is the particle image diameter without streaking. The value of d_τ was fixed at three pixels in this study. Standard PIV algorithms assume that the particles are circular ($d_x = d_y$), in which case the cross-correlation functions are circular. Streaked particle images ($d_x > d_y$), however, lead to noncircular correlations, which is problematic because there is increased uncertainty in locating the peak of the correlation along the axis of greatest elongation.

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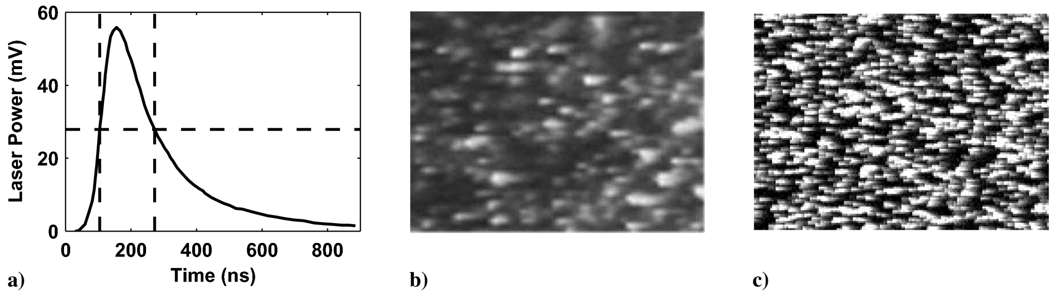

Fig. 1 Effect of laser pulse duration on particle images: a) laser pulse width profile,[1] b) measured particle image[1] revealing the comet-shaped profile of the particles, and c) synthetic PIV image revealing a similar asymmetry in the particle image.

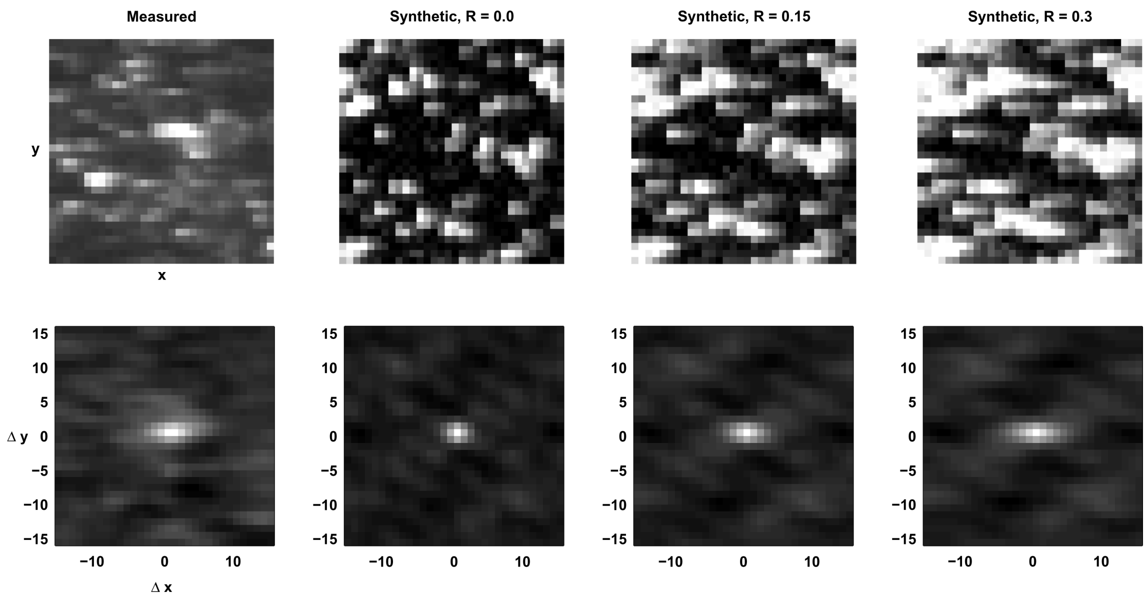

Fig. 2 Measured and synthetic images used to check the effect of particle image streaking on PIV accuracy: particle images (top) and cross-correlation maps (bottom). A measured particle image from Bueno et al.[1] is at the upper left, and its cross-correlation function is shown below it. Simulated asymmetric particle images for various ratios R and the corresponding correlation maps are also shown. The particle displacement used was 14 pixels.

The shape of the laser pulse width would also dictate the streaky nature of the particles. The shape of the laser pulse width does not influence the measurements that use lasers with small pulse widths ($\tau \sim 10$ ns). However, the laser pulse width plays an important role when the values of τ are larger. For example, the laser pulse width of a Nd:YLF laser (Coherent, Inc., Evolution 90) was measured using a fast photodiode, and the resulting time trace is shown in Fig. 1. The figure shows that the pulse has a width of about 200 ns (full width at half the maximum value) with a rapidly rising edge and a slowly falling tail. A sample particle image, acquired in a Mach 2 flow with the same Nd:YLF laser,[1] is also shown in Fig. 1. The figure shows that the streaks are in fact "comet shaped" with the tail extending in the direction of flow motion. This particular shape is the result of the asymmetric shape of the laser pulse. Therefore the asymmetry of the particle streaks might also be an important parameter in the errors in pixel displacements computed. Figure 1 also shows an example of a synthetic particle image generated with asymmetric streaks. This asymmetry was achieved computationally by using only one-half of the Gaussian profile given in Eq. (4) along the x direction.

Synthetic PIV image pairs were generated for various values of R and ΔX to study the effect of particle streaking on PIV results. Two separate data sets for the asymmetric and symmetric Gaussian profiles of the particle images were generated. However, further investigation of the cross-correlation maps revealed that the asymmetry of the particle profile did not affect the estimation of pixel displacement because correlation maps did not appear to be different between the two cases. Also, the cross-correlation map of the measured particle image (shown in Fig. 2) does not appear to be different from a cross-correlation map computed from a particle image with symmetric profiles. All results presented in this Note are from the asymmetric particle profile data set (because we wanted to match the artificial particle images to the measured particle images). However, differences in the results cannot be ruled out if a particle profile different from the Gaussian profile is used.

Values of R ranging from 0.0 to 0.3 and a displacement range of 2–40 pixels were considered in the study. Only integral values of pixel displacements were considered, and hence this study cannot address possible pixel locking effect of particle streaking. The value R would be almost zero for most PIV applications in low-speed flows where the pulse separations are orders of magnitude higher than the pulse duration. In comparison, the cinematographic PIV in Mach 2 flow of Bueno et al.[1] has a value of $R \approx 0.15$ ($\tau = 200$ ns and $\Delta t = 1.5$ μs) and $\Delta X \sim 14$–18 pixels.

Figure 2 shows the synthetic particle images for various values of R for a fixed particle displacement of 14 pixels. The figure also reveals a portion of the experimentally acquired PIV image in a supersonic flow,[1] where $R \sim 0.15$, $\tau \sim 200$ ns, and $\Delta t = 1.5$ μs). The measured particle image contains particles with various aspect ratios depending on the local fluid velocity. The uncertainty in the displacement computed from the particles of different aspect ratios can be computed by processing the synthetic PIV images with the conventional PIV algorithm that is used to process the experimental PIV data.

A series of 10 images (of size 512×256 pixels) were generated for each combination of displacement ΔX and ratio R. A window

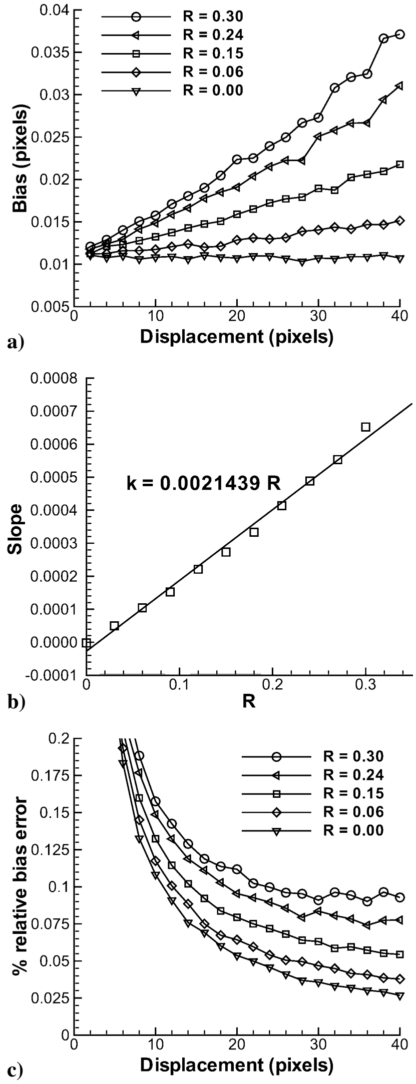

Fig. 3 Effect of particle streaks on bias error: a) bias error computed using CDI technique,[4] b) variation of the slope of bias error vs displacement with *R*, and c) percentage relative bias error in pixel displacement for various ratios.

of size 32 × 32 pixels (50% overlap) was used to compute the displacement fields, which resulted in a total of 465 vectors per image and a total of 4650 vectors for every $\Delta X - R$ combination. Displacement fields were computed with an in-house MATLAB® based software that uses the central-difference-iterative (CDI) technique[4] with three-iterations (128 × 128 to 64 × 64 and finally 32 × 32).

Figure 3a shows the rms of the bias error (defined as the difference between the measured displacement and actual displacement) as a function of pixel displacement for several values of R. The curves indicate that the bias error increases with increasing value of R for any given pixel displacement ΔX. The maximum bias error is approximately 0.035 pixels for $R = 0.3$ and $\Delta X = 40$ pixels. The curve for $R = 0$ is the standard bias error associated with circular particles, and its value is approximately equal to 0.01 pixels. Therefore, particle streaking increases the bias error by over three times for $R = 0.3$ at higher displacements. Figure 3a also seems to suggest that the bias error varies almost linearly with increasing displacement in the form

$$b = k\Delta X + C \tag{6}$$

where C is the intercept at $\Delta X = 0$ whose value is approximately 0.015 pixel for all values of R. The data in Fig. 3a can be fitted to a straight line, and the slope k can be recorded for a range of values of R. Figure 3b shows the variation of slope k with the value of R. The value of k increases linearly with increasing R. The slope of this linear increase is approximately 0.0021. Therefore for a given value of R and mean pixel displacement the value of the bias error as a result of particle streaking can be found using Fig. 3b and Eq. (6).

Figure 3c reveals the relative percentage error for various displacements and various ratios. This shows that the relative effect of the error decreases with increasing displacement. The eccentricity of the particle image ($e = d_x/d_y$) is directly proportional to the pixel displacement, and therefore the bias error will be higher for particle images with larger eccentricities.

Although the bias error caused by particle streaking might seem relatively insignificant, the presence of bias will increase the uncertainty in any velocity gradient measurements (For example, velocity fields accurate to within 1–2% could lead to a 10–20% uncertainty in the gradient; see Ref. 5). Also, these results show the errors computed in synthetic images with uniform pixel displacement across the image. In real flows, the interrogation windows would include velocity gradients, intensity gradients, and other factors that would increase the bias error of the measurement. Therefore, under experimental circumstances it is necessary to minimize the effect of particle streaking on the bias error to obtain quality measurements. Ideally, the pixel displacements should be small to decrease the eccentricity of the particle images. However, that is not a feasible solution because the relative error in the velocity measured would be high. Figure 3c shows that the relative effect of the error decreases with increasing displacement, suggesting that the pixel displacements in the experiment should be large enough to overcome the bias error. In practice, it is probably not possible to achieve pixel displacements much greater than 20 pixels in most flows, depending on the field of view and magnification. Therefore there is a need to strike a balance between the pixel displacement and the eccentricity of the particle (because large displacements leads to particle images with larger eccentricity). This balance could be achieved by maximizing pulse separation Δt (which would decrease the ratio R), increasing the pixel displacement, and hence reducing the relative bias error associated with the measurement.

Summary

The effect of laser pulse duration on the bias error in particle image velocimetry is investigated using synthetic images. Long laser pulse duration results in particle streaks along the predominant flow direction. The presence of particle streaks increases bias error and consequently increases the uncertainty in velocities and velocity gradients. Particle streak length is directly proportional to the ratio of pulse duration to pulse separation R and the pixel displacement in the predominant flow direction. An algebraic expression for the bias error as a function of R and ΔX is derived. The bias error varies almost linearly with increasing pixel displacement for a given value of R. The bias error also increases with increasing value of R for any given pixel displacement. This suggests that the pulse separation between lasers must be much larger than the pulse duration of a laser to minimize the effect of particle streaking on bias error.

Acknowledgments

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